

Record statistics of financial time series and geometric random walks

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The study of record statistics of correlated series is gaining momentum. In this work, we study the records statistics of the time series of select stock market data and the geometric random walk, primarily through simulations. We show that the distribution of the age of records is a power law with the exponent α lying in the range $1.5 \leq \alpha \leq 1.8$. Further, the longest record ages follow the Fréchet distribution of extreme value theory. The records statistics of geometric random walk series is in good agreement with that from the empirical stock data.

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I. INTRODUCTION

In popular parlance, records are associated with record breaking events. Common examples include extreme weather events such as the occurrence of lowest or highest temperatures [1], unparalleled sport performances in Olympic and other events [2], financial downturns like the major stock market crashes. In recent years, there is an increasing interest in the study of record statistics in the context of global warming and climate change [3], occurrence of cyclones and floods [4] and stock markets. In physics, records statistics is useful in understanding the behavior of stochastic motion of a domain wall in metallic ferromagnetic materials [5] and as an alternative indicator of quantum chaos in kicked rotor model [6]. Even as the record breaking events continue to enjoy media attention, there is also an increased research interest in the statistical study of record events [7–12].

For a discretely sampled stochastic time series $x_t, t = 1, 2, 3, \dots, N$, record events are those that are larger (smaller) than all the preceding events. An event at $t = T$ would be an upper record if $x_T > \max(x_1, x_2, \dots, x_{T-1})$. Then, some of the relevant questions of interest are the probability for the occurrence of record at any given time, mean number of records in a certain time window and record age, *i.e.*, how long a record is expected to survive. The result for most of these questions is known for uncorrelated random variables [13]. However, it is known that most of the physically observed time series, *e.g.*, temperature, stock market volatility, earthquake magnitudes, are strongly correlated [14]. The record statistics for such cases is beginning to receive research attention.

Recently, the record statistics of correlated series such as the positions of random walker was studied [7, 10, 11]. Random walk is a fundamental model in physics and has applications in many areas including the dynamics of stock markets. It was shown that if the increments of the random walker are drawn from a continuous and symmetric function $\phi(\xi)$, then the mean number of records,

for large N , is proportional to \sqrt{N} and the mean record age $\langle r \rangle \propto N$ [7]. These results have further been generalized to the case of random walk with a constant drift with application to stock market data [15] and also to multiple random walkers [11].

In spite of such growing interest in correlated series, very few works have focussed on the records statistics in empirical stock data [11, 12, 16]. In this paper, we report on the record statistics of empirical stock data to understand two quantities of interest not studied earlier, namely, (i) the distribution of record age and (ii) the distribution of longest record ages. We present our analysis of stock data in the context of geometric random walk model, which is considered as one of the suitable models for the dynamics of stock data [17]. In addition, it must be pointed out that GRW has other applications as well, including as a model for interacting neurons [18].

In this paper, we analyze the upper record statistics for 18 stocks, for which longest data is available in the public domain. The data used in this work is described in the Appendix. Most aspects of record statistics, especially quantities such as the mean number of records, record age distribution, longest record age etc., depend only on the position of record breaking event on time axis and *not* on its magnitude. We study these quantities using geometric random walk as the benchmark model. We show that both for the records in stock data and geometric random walk series the distribution of record age r is consistent with $P(r) \sim r^{-\alpha}$, with the exponent $1.5 \leq \alpha \leq 1.8$ and the longest records r_{max} fall in the class of type-II generalized extreme value (Fréchet) distribution.

II. DISTRIBUTION OF RECORD AGES

Geometric random walk (GRW) has not attracted as much attention as the random walk model except in the context of financial applications [17]. GRW model is given by,

$$y_{i+1} = y_i \exp(\xi_i), \quad i = 1, 2, 3, \dots, N. \quad (1)$$

In this, ξ_i is Gaussian distributed $G(\mu, \sigma)$ with mean μ and standard deviation σ . This implies that the 'log

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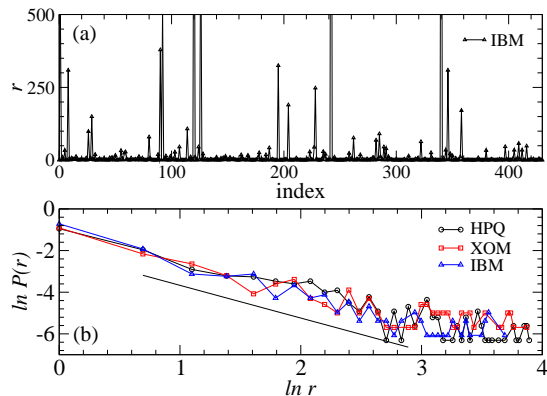


FIG. 1. (Color online) (a) Record ages (in days) calculated from IBM stock data. (b) The distribution of record ages for three stocks. The best fit solid line in (b) has slope -1.58 ± 0.15 .

returns' $R_i = \log(y_{i+1}/y_i)$ are also Gaussian. The log-returns from the empirical stock data is known to be approximately Gaussian distributed over a wide range of timescales [17].

Record age is the time duration r between two successive occurrences of a record, *i.e.*, the time for which a record survives. Record age distribution will provide insights into how long a record can be expected to survive and is useful in hazard estimation problems. Though the mean record age has been analytically determined for random walk problems in earlier works [7, 15], there have been no results for the distribution of record age.

In Fig. 1(a), we show the record ages obtained from IBM stock data. In this, record ages longer than 500 are not shown since they mask the details near $r = 1$. The longest record age (not visible in Fig. 1(a)) is 2313 days and the shortest is 1 day. Thus, in this case, the record ages vary over 3 orders of magnitude. Clearly, they depend on the length N of data being considered since the longest record age cannot exceed the length of data. Fig. 1(b) displays the distribution of record age computed from the stock prices of three stocks (HPQ, XOM and IBM) with the longest available time series. In log-log plot shown in this figure, the distribution, for most part, is consistent with a power law of the form

$$P(r) \sim A r^{-\alpha} \quad (2)$$

with the exponent $\alpha \approx 1.58$ and A being the normalization constant that can be written in terms of harmonic number $H_{N,\alpha}$. However, the tail of computed distribution flattens out due to effect arising from the finite size of the data. In order to improve the statistics, we use GRW simulations (Eq. 1) with ξ_i drawn from normal distribution with parameters values $\mu = \langle \mu_{emp} \rangle = 0.00031$ and $\sigma = \langle \sigma_{emp} \rangle = 0.015$. These parameter values μ_{emp} and σ_{emp} were computed from the empirical stock data by averaging over the individual values of μ and σ obtained for each stock. The record age distribution for each value of N , shown in Fig. 2(a), is averaged over 10^5 GRW re-

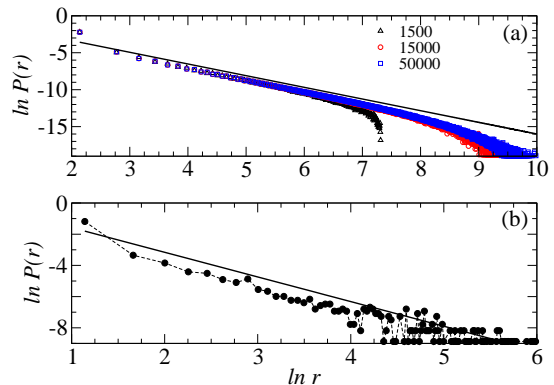


FIG. 2. (Color online) The distribution of record ages obtained from (a) GRW simulations for three values of N and (b) stock data other than those shown in Fig. 1(b). See text for details. The solid line in (a) has slope -1.652 ± 0.006 and in (b) has slope -1.611 ± 0.051 .

alizations. Clearly, the distribution in Fig. 2(a) can be represented as a power law in Eq. 2 with the exponent $\alpha = 1.652 \pm 0.006$. Significantly, it is independent of the value of N . In contrast to quantities like the mean number of records which depend on N [7, 12, 15], the distribution of record ages is characteristic statistical property of record breaking events independent of the length of data. Further, as N increases, the range over which the power law is valid also increases implying that the tail behavior is a finite size effect. Within the parametric regime relevant for the stocks listed in the Appendix, namely, $0.0001 \leq \mu \leq 0.0005$ and $0.01 \leq \sigma \leq 0.05$, we did not find any systematic relation between these parameters and the exponent α .

Based on the results displayed in Fig. 1(b), we might expect that all the individual stocks will display nearly the same value of α even if N is different for each one of them. Indeed, the value of the exponent lies in the range $1.5 \leq \alpha \leq 1.8$ for the stocks listed in Appendix. Hence, we combined the record ages computed from the rest of stock data in Appendix (other than HPQ, XOM and IBM) and the resulting distribution is displayed in Fig. 2(b). The power law form (Eq. 2) is seen in the figure with a value of exponent $\alpha \approx 1.611 \pm 0.051$.

III. LONGEST RECORD AGE

Given that the record age is distributed as a power law, it is of interest to understand the distribution of longest record age. Clearly, shortest record age cannot be less than unity, a restriction arising from the resolution of the data measurement. Similarly, any record age longer than the length of the time series N cannot be resolved. In ref. [7, 12], it was pointed out that for a symmetric random walk process, the longest record age is proportional to N . However, the distribution of longest record age has not been discussed earlier.

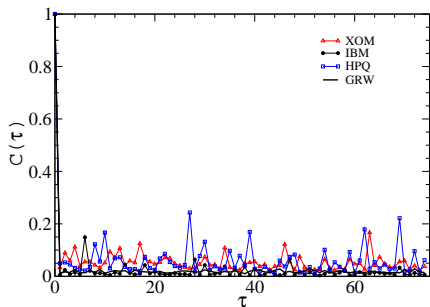


FIG. 3. (Color online) The autocorrelation function of the record ages obtained from three different stock data and GRW simulations. For GRW results, we have used the same value of N as for IBM stock, $\langle\mu_{emp}\rangle = 0.00031$ and $\langle\sigma_{emp}\rangle = 0.015$. See text for details.

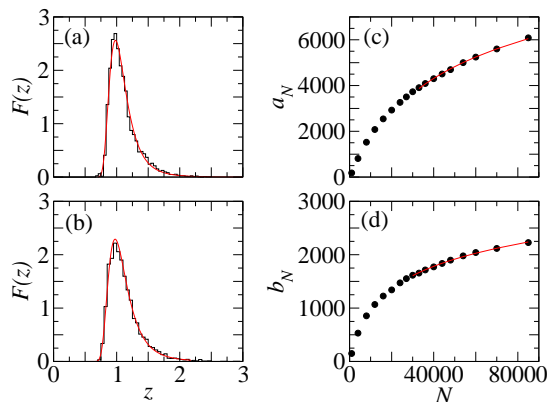


FIG. 4. (Color online) Scaled distribution of longest record age obtained from GRW simulations with (a) $N = 15000$ and (b) $N = 85000$. The solid curve is the Fréchet distribution with shape parameter $k > 0$. (c) The location parameter a_N and (d) the scale parameter b_N of the Fréchet distribution shown as a function of N . The solid line in (c,d) is the logarithmic fit for $N > 30000$.

In this section, we show that the longest record age falls in the class of type-II generalized extreme value distribution, namely, the Fréchet distribution [19]. First clue for this result arises from the record ages that are uncorrelated, to a good approximation. Fig. 3 shows the autocorrelation function $C(\tau) = \langle x_t x_{t+\tau} \rangle$ for the stock data. It reveals that the record ages are, at best, weakly correlated. The record ages obtained from GRW simulations (with parameters $\mu = \langle\mu_{emp}\rangle, \sigma = \langle\sigma_{emp}\rangle$ chosen similar to that in Fig. 2(a)) also show a similar behavior. For such fast decay of correlations, extreme value theory for independent variables holds good [20]. Hence, we can expect the longest record age to follow the generalized extreme value distributions.

Fig. 4(a,b) shows the distribution of longest record age r_{max} , in terms of the scaled variable $z = 1 + k(r_{max} - a_N)/b_N$, for the GRW simulations with parameters same as for Fig. 2(a). In this, a_N and b_N are location and scale parameters dependent on N . This figure reveals a

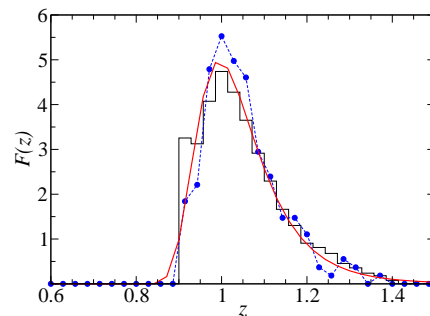


FIG. 5. (Color online) The distribution of scaled longest record ages computed from stock data (solid circles), GRW simulations (histogram) with $N = 1000$. The solid curve is the Fréchet distribution with parameters a_N and b_N corresponding to $N = 1000$.

good agreement with the Fréchet distribution [19]

$$F(z) = \frac{1}{b_N} z^{-1-1/k} e^{-z^{-1/k}}, \quad (z > 0), \quad (3)$$

with shape parameter $k > 0$. This is the extreme value distribution consistent with results shown in Figs. 1,2, *i.e.*, the distribution of record ages $P(r)$ has a lower end cut-off and its tail decays as a power law. The agreement with Fréchet distribution gets better for $N \gg 1$. The dependence of the location parameter a_N and the scale parameter b_N on N shown in Fig. 4(c,d) reveals that $\ln N$ function provides a good representation of the data for $N > 30000$. Using this fit and the mean of Fréchet distribution $\langle z \rangle = a_N + (b_N/k)(\Gamma(1-k) - 1)$, we get the asymptotic mean of the longest record ages as $\langle r_{max} \rangle \propto \ln N$. This is the result obtained analytically in Ref. [12] without using extreme value theory.

Finally, we compute the distribution of longest record ages from the stock data. To circumvent the shortage of data, we divided the empirical stock data into windows of length $N = 1000$. The longest record age from each of these windows was tabulated for each stock. All such data of extreme record ages from all the stocks were combined together to compute the (scaled) distribution shown in Fig. 5 as solid circles. The histogram in the figure is obtained from 10^5 ensemble GRW simulations with $N = 1000$ and other parameters chosen as done in Fig. 2. The solid curve is the Fréchet distribution with $k > 0$. The distribution $F(z)$ computed from stock data displays a reasonable agreement with Fréchet distribution. The deviations could partly be attributed to the insufficient stock market data to compute extreme record ages. We must also point out that both GRW simulations and stock data display pronounced deviation from Fréchet distribution for $z < 1$.

IV. SUMMARY AND DISCUSSIONS

In summary, we have analyzed the stock data for two quantities of interest in the study of record statistics,

namely, the distribution of record ages and the longest record age. The results have been obtained based on the analysis of 18 stocks for which the data is available in the public domain. We also study the geometric random walk series as a suitable reference model in the context of the time series of stocks. For the stock data and the GRW simulations, the record ages are distributed as a power law with exponent in the range $1.5 \leq \alpha \leq 1.8$. The record ages are uncorrelated, to a good approximation. The longest record ages are well described by the Fréchet distribution of the extreme value theory.

The results presented in this work also applies to the records statistics of the positions of a standard random walker. This is possible because the random walk and GRW are related through a simple time-independent transformation. The record age distribution $P(r)$ is independent of N to within the numerical errors and it does not preclude the mean record age from being dependent on N [12]. Record ages being nearly uncorrelated implies that predicting the length of time before the occurrence of next record event based on historical data is unlikely to be easy even though the mean record age can be determined [7]. The longest record age is Fréchet distributed for $N \gg 1$ and pronounced deviations exist for small N . While an analysis of longer and bigger portfolio of stock data will yield better estimates for power law exponent α and also for the longest record age distribution, it would be interesting to analytically obtain these results.

Appendix: Data used in the analysis

In this work, we use the daily closing values, corrected for splits and dividends, of the following stocks. These are publicly accessible from finance.yahoo.com. Standard stock symbols are used to indicate stock names.

Stock	Years	Length of data	Stock Exchange
IBM (IBM)	1962-2012	12764	NYSE
GIS (General Mills Inc.)	1983-2012	7358	NYSE
AAPL(Apple Inc.)	1984-2012	7067	NASDAQ
XOM (Exxon Mobil Inc.)	1970-2012	10777	NYSE
FP.PA (Total SA)	2000-2012	3435	PARIS
GD (General Dynamics Co.)	1977-2012	8600	NYSE
GE (General Electric Co.)	1962-2012	12764	NYSE
HPQ (Hewlett-Packard Co.)	1962-2012	12747	NYSE
NTT (Nippon Telegraph ...)	1994-2013	4656	NYSE
SNP (China Petroleum and ...)	2000-2013	3124	NYSE
TM (Toyota Motor Co.)	1993-2013	5021	NYSE
VOW.DE (Volkswagen AG)	2000-2013	3423	XETRA
CVX (Chevron Co.)	1970-2013	10910	NYSE
WMT (Walmart Stores Inc.)	1972-2013	10238	NYSE
F (Ford Motors)	1977-2013	9141	NYSE
COP (ConocoPhillips)	1982-2013	7878	NYSE
BRK.A (Berkshire Hathaway)	1990-2013	5825	NYSE
BP (BP plc)	1988-2013	6445	NYSE

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- [1] Christina DeConcini and C. F. Tompkins, World Resources Institute Fact Sheet (2013). www.wri.org/publication/fact-sheet-2012-year-record-breaking-extreme-weather-and-climate
- [2] J. J. Hernandez Gmez, V. Marquina and R. W. Gomez, Eur. J. Phys. **34**, 1227 (2013); See also www.wired.com/playbook/2012/08/physics-long-jump-linear-regression ; D. Gembris, J. G. Taylor and D. Suter, J. Appl. Stat. **34**, 529 (2007).
- [3] S. Rahmstorf and D. Coumou, PNAS **108**, 17905 (2011).
- [4] T. C. Peterson, M. P. Hoerling, P. A. Stott and S. C. Herring, Bulletin. Am. Meteo. Soc. (Special Supplement) **94**, S1 (2013).
- [5] B. Alessandro, C. Beatrice, G. Bertotti and A. Montorsi, J. Appl. Phys. **68**, 2901 (1990).
- [6] Shashi C. L. Srivastava, A. Lakshminarayan and Sudhir Jain, EPL **101**, 10003 (2013).
- [7] S. N. Majumdar and R. M. Ziff, Phys. Rev. Lett. **101**, 050601 (2008).
- [8] G. Wergen, J. Phys A **46**, 223001 (2013); J. Krug, J. Stat. Mech. Theory and Experiment P07001 (2007); I. Eliazar and J. Klafter, Phys. Rev. E **80**, 061117 (2009); J. Franke, G. Wergen and J. Krug J. Stat. Mech. Theory and Experiment P10013 (2010); E. Ben-Naim and P. L. Krapivsky, Phys. Rev. E **88**, 022145 (2013).
- [9] C. M. Row and L. E. Derry, Geophys. Res. Lett **39**, (2012); W. I. Newman, B. D. Malamud and D. L. Turcotte **82**, 066111 (2010); G. Wergen and J. Krug, EPL **92**, 30008 (2010).
- [10] Y. Edery, A. B. Kostinski, S. N. Majumdar and B. Berkowitz, Phys. Rev. Lett. **110**, 180602 (2013); S. Sabhapandit, EPL **94**, 20003 (2011); S. N. Majumdar, Physica A **389**, 4299 (2010); C. Godreche, S. N. Majumdar and G. Schehr, J. Phys. A **47**, 255001 (2014).
- [11] G. Wergen, S. N. Majumdar and G. Schehr, Phys. Rev. E **86**, 011119 (2012).
- [12] S. N. Majumdar, G. Schehr and G. Wergen, J. Phys. A **45**, 355002 (2012).
- [13] V. B. Nevzorov, *Records : Mathematical Theory*, (Americal Mathematical Society, 2001); B. Schmittmann and R. K. P. Zia, Am. J. Phys. **67**, 1269 (1999).
- [14] P. Doukhan, G. Oppenheim and M. S. Taqqu, *Theory and Applications of Long-Range Dependence*, (Springer, 2003).
- [15] G. Wergen, M. Bogner and J. Krug, Phys. Rev. E **83**, 051109 (2011).
- [16] G. Wergen, Physica A **396**, 114 (2014).
- [17] David Ruppert, *Statistics and Data Analysis for Financial Engineering*, (Springer, New York, 2011); Frank J. Fabozzi, *Encyclopedia of Financial Models*, (Wiley, 1st edition, 2012).
- [18] R. Kuhn and P. Neu, J. Phys. A **41**, 324015 (2008).
- [19] S. Coles, *An Introduction to Statistical Modeling of Extreme Values*, (Springer, 2001).
- [20] M. R. Leadbetter and H. Rootzen, Ann. Probab. **16**, 431 (1988).